



***Gainesville State College  
Sixteenth Annual Mathematics Tournament  
April 10, 2010***

***Morning Component***

*Good morning!*

*Please do NOT open this booklet until given the signal to begin.*

*There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.*

*The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 36 will be used again as a tie-breaker.*

*This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.*

*You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.*

*Good luck!*

***Do Not Open Until Signaled.***

**Sixteenth Annual Gainesville State College  
Mathematics Tournament**

*You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.*

1. Find the limit:  $\lim_{x \rightarrow 0} \frac{10x^2}{1 - \cos x}$ .

- a)  $\infty$
- b) 20
- c) 15
- d)  $e^{-10\pi}$
- e) none of the above

2. Consider the function  $f(x) = \sqrt{x^2 + \sqrt{g(x)}}$ . Given that both  $f$  and  $g$  are differentiable functions of  $x$  and that  $g\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{4}$ , find the exact value of  $g'\left(\frac{1}{\sqrt{2}}\right)$  if  $f'\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$ .

- a) 0
- b)  $\sqrt{2}$
- c)  $\sqrt{\frac{1}{2} + \sqrt{2}}$
- d)  $-\frac{1}{\sqrt{2}}$
- e) none of the above

3. The area of a circular oil spill is increasing at the rate of 40 square kilometers per hour. At what rate is the radius of the spill increasing, when the radius is 4 kilometers?

a)  $\frac{\pi}{5} \text{ km/hr}$

b)  $\frac{5}{\pi} \text{ km/hr}$

c)  $10\pi \text{ km/hr}$

d)  $\frac{10}{\pi} \text{ km/hr}$

e) none of the above

4. Find  $\int \arcsin(3x) dx$ .

a)  $\arcsin(3x) + C$

b)  $\frac{3}{\sqrt{1-9x^2}} + C$

c)  $x \arcsin(3x) + \frac{3}{\sqrt{1-9x^2}} + C$

d)  $x \arcsin(3x) + \frac{\sqrt{1-9x^2}}{3} + C$

e) none of the above

5. A wire of length  $L$  is cut into two pieces. One piece, of length  $x$ , is bent to form a circle, and the other to form a square. Find  $x$  so that the sum of the areas of the circle and the square is a minimum.

a)  $x = \frac{L}{2 + \pi}$

b)  $x = \frac{\pi L}{4 + \pi}$

c)  $x = \frac{\pi^2 L}{(2 + \pi)^2}$

d)  $x = \frac{\pi L}{2 + \pi}$

e) none of the above

6. If  $c > 0$  and  $f(x) = e^x - cx$ , then the minimum value of  $f$  is

a)  $f(c)$

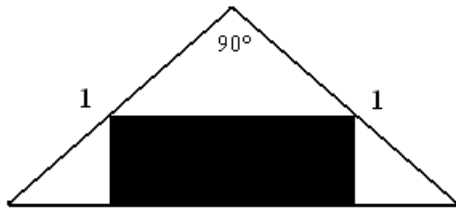
b)  $f(e^c)$

c) no minimum exists

d)  $f(\ln c)$

e) none of the above

7. Find the maximal area of a rectangle inside a right isosceles triangle with legs of length 1 as indicated in the picture below:



- a)  $\frac{1}{2}$
- b)  $\frac{\sqrt{2}}{2}$
- c)  $\frac{1}{3}$
- d)  $\frac{1}{4}$
- e) none of the above
8. Find the limit:  $\lim_{x \rightarrow 1} \left(\frac{1}{x}\right)^{1/\ln(x)}$ .

- a)  $e^{-1}$
- b) 1
- c)  $e$
- d) 0
- e) none of the above

9. If  $y = x^3 + x$  and  $u = y^2 - 1$ , find  $\frac{dx}{du}$ .

a)  $\frac{1}{2(x^3 + x)(3x^2 + 1)}$

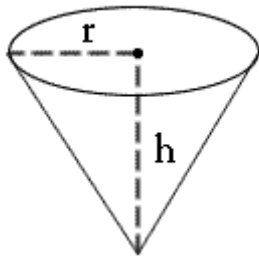
b)  $\frac{1}{2(3x^2 + 1)}$

c)  $\frac{1}{2(x^3 + x)}$

d)  $2(x^3 + x)(3x^2 + 1)$

e) none of the above

10. Water is poured into a conical cup (see picture below) at the rate of  $\frac{2}{3}$  cubic inches per second. If the cup is 6 inches tall and if the top of the cup has a radius of 2 inches, how fast is the water level rising, when the water is 4 inches deep?



a)  $\frac{3}{8\pi} \text{ in/sec}$

b)  $\frac{2\pi}{3} \text{ in/sec}$

c)  $\frac{2}{3\pi} \text{ in/sec}$

d)  $\frac{1}{3}\pi \text{ in/sec}$

e) none of the above

11. Let  $f(x) = (x-2)^2 + 1$  and  $g(x) = x-1$  over the interval  $[1, 3]$ . Find the volume obtained by rotating the area between the curves around the  $y$ -axis.
- a)  $4\pi$
  - b)  $3\pi$
  - c)  $2\pi$
  - d)  $\pi$
  - e) none of the above
12. Evaluate  $\int_{-1}^1 (\sin(x^3) + x^2) dx$ .
- a) 0
  - b)  $\frac{2}{3}$
  - c)  $\frac{1}{3}$
  - d) undefined
  - e) none of the above
13. Find an equation of the parabola  $y = ax^2 + bx + c$  that passes through  $(0, 1)$  and is tangent to the line  $y = x - 1$  at  $(1, 0)$ .
- a)  $y = x^2 - 2x + 1$
  - b)  $y = 2x^2 - 3x + 1$
  - c)  $y = 3x^2 - 4x + 1$
  - d)  $y = 4x^2 - 5x + 1$
  - e) none of the above

14. Suppose  $f$  is an even function differentiable for all real numbers. What can be said about the derivative of the function at  $x = -5$ ?

- a)  $f'(-5) = f'(5)$
- b)  $f'(-5) = 0$
- c)  $f'(-5) = -f'(5)$
- d)  $f'(-5) = \frac{1}{2}f'(10)$
- e) none of the above

15. Find the limit:  $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$ .

- a) 0
- b) -1
- c)  $\infty$
- d) 1
- e) none of the above

16. The derivative of the function  $f(x) = e^x x^e$  is:

- a)  $x^e e^x + x^{e+1} e^{x+1}$
- b)  $x^e e^x + x^{e-1} e^{x+1}$
- c)  $x^e e^x + x^{e-1} e^{x-1}$
- d)  $x^{e+1} e^{x-1}$
- e) none of the above

17. Find a second degree polynomial  $p(x)$  such that  $xp''(x) + p(x) = x^2 + 1$ .

- a)  $x^2 + x + 1$
- b)  $-2x^2 + x + 1$
- c)  $x^2 + x$
- d)  $x^2 - 2x + 1$
- e) none of the above



18. Find the volume of the solid formed by revolving the region bounded by the graphs of  $x = \sqrt{y-1}$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$  about the  $y$ -axis.

a)  $\frac{\pi}{2}$

b)  $\pi$

c)  $\frac{3\pi}{2}$

d)  $2\pi$

e) none of the above

19. Evaluate  $\int_1^{4/3} \frac{y^4 + y^2 - 1}{y^3 + y} dy$ .

a)  $\frac{7}{18} + \ln\left(\frac{5}{4\sqrt{2}}\right)$

b)  $\frac{7}{9} + \ln\left(\frac{5}{4\sqrt{2}}\right)$

c)  $\frac{7}{9} + \ln\left(\frac{4}{5\sqrt{2}}\right)$

d)  $\frac{7}{18} + \ln\left(\frac{4}{5\sqrt{2}}\right)$

e) none of the above

20. Find the constants  $a$  and  $b$  such that the following function is continuous on the entire real number line.

$$f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ ax + b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \geq 3 \end{cases}$$

- a)  $a = -1, b = -1$   
b)  $a = -1, b = 1$   
c)  $a = 1, b = -1$   
d)  $a = 1, b = 1$   
e) none of the above
21. Suppose that the functions  $f$  and  $g$  are defined throughout an open interval containing the point  $x_0$ ,  $f$  is differentiable at  $x_0$ ,  $f(x_0) = 0$ , and that  $g$  is continuous at  $x_0$ . What can be said about the differentiability of the product  $fg$  at  $x_0$ ?

- a)  $fg$  is differentiable at  $x_0$  only if the function  $g$  is differentiable at  $x_0$ .  
b)  $fg$  is differentiable at  $x_0$  no matter if the function  $g$  is differentiable at  $x_0$  or not.  
c)  $fg$  is differentiable at  $x_0$  only if the functions  $f$  and  $g$  satisfy the condition  $g(x) \leq f(x)$  near  $x_0$ .  
d)  $fg$  is differentiable at  $x_0$  only if  $g(x_0) = 0$ .  
e) none of the above

22. Find the limit:  $\lim_{t \rightarrow 0} \left[ \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right]$ .

- a) 0  
b)  $-\frac{1}{16}$   
c) Does not exist  
d)  $-\frac{1}{2}$   
e) none of the above

23. Find the derivative of the function  $f(x) = (\sin x)^x$  for  $x$  in the interval  $(0, \pi)$ .

a)  $(\sin x)^x \left( \ln(\sin x) + \frac{x \cos x}{\sin x} \right)$

b)  $x(\sin x)^{x-1} \cos x$

c)  $(\sin x)^x \left( \ln(\sin x) + \frac{x}{\sin x} \right)$

d)  $\ln(\sin x) + \frac{x \cos x}{\sin x}$

e) none of the above

24. Find  $\int x^4 e^{x^5} dx$ .

a)  $\frac{1}{5} x^5 e^{x^5} + C$

b)  $\frac{1}{5} x^5 e^{x^5+1} + C$

c)  $\frac{1}{5} e^{x^5} + C$

d)  $\frac{1}{5} e^{x^5+1} + C$

e) none of the above

25. Find the volume  $V$  of the solid generated by revolving the first quadrant region bounded by the parabola  $y = 4 - x^2$ ,  $x = 0$ , and  $y = 0$  about the line  $x = 2$ .

a)  $\frac{40}{3} \pi$

b)  $\frac{47}{2} \pi$

c)  $\frac{18}{7} \pi$

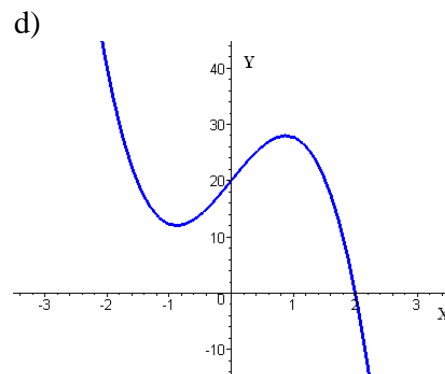
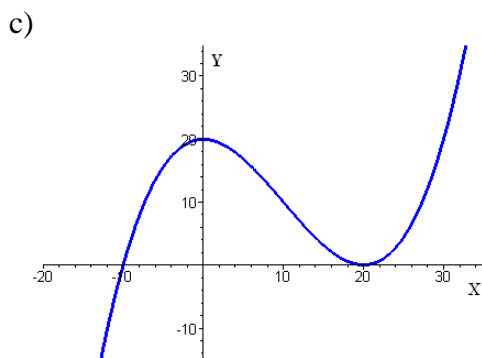
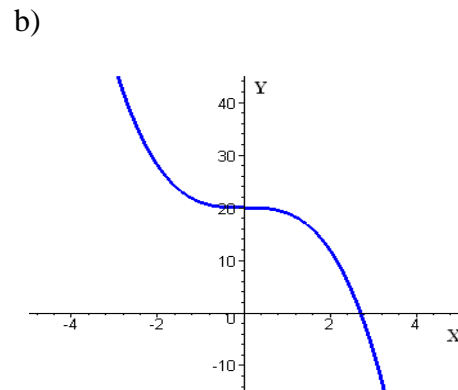
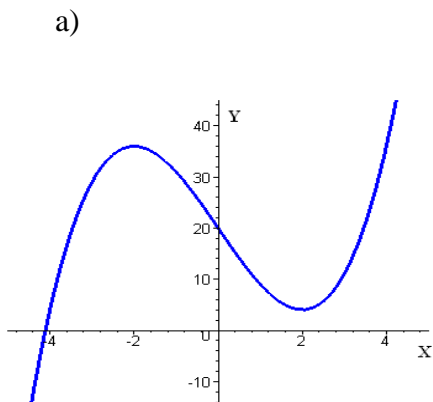
d)  $40\pi$

e) none of the above

26. Evaluate  $\int_1^{4/3} \sqrt{\frac{x-1}{x^5}} dx$

- a)  $\frac{1}{6}$
- b)  $\frac{1}{12}$
- c)  $\frac{1}{24}$
- d)  $\frac{1}{36}$
- e) none of the above

27. Which of the following is the correct sketch of the graph of the function  $f(x) = x^3 - 12x + 20$



- e) none of the above

28. Find  $\frac{d^{999}}{dx^{999}}(\sqrt{1-\sin^2 x})$  for  $x$  in the interval  $[\pi/2, \pi]$ .

- a)  $\sin x$
- b)  $\cos x$
- c)  $-\cos x$
- d)  $-\sin x$
- e) none of the above

29. Find the limit:  $\lim_{x \rightarrow 2^+} \left( \frac{1}{\ln(x-1)} - \frac{1}{x-2} \right)$ .

- a) 0
- b)  $\frac{1}{2}$
- c) 1
- d)  $\infty$
- e) none of the above

30. If  $f(x) = \int_0^{1/x} \frac{1}{t^2+1} dt + \int_0^x \frac{1}{t^2+1} dt$ , then

- a)  $f'(x) = \frac{2}{x^2+1}$  for  $x > 0$
- b)  $f(x) = \tan^{-1} x$  for  $x > 0$
- c)  $f'(x) = 1$  for  $x > 0$
- d)  $f(x) = \text{constant}$  for  $x > 0$
- e) none of the above

31. Find  $\int \frac{dx}{1 + \sin x}$ .

- a)  $\tan x - \sec x + C$
- b)  $\ln|\sin x + 1| + C$
- c)  $\frac{\sin x + 1}{\cos x} + C$
- d)  $\ln|1 - \sin x| + C$
- e) none of the above

32. Logs with a circular cross section 4.0 ft in diameter are cut in half lengthwise. Find the largest rectangular cross-sectional area that can be cut from one of the halves.

- a)  $2.0 \text{ ft}^2$
- b)  $4.0 \text{ ft}^2$
- c)  $4\pi \text{ ft}^2$
- d)  $8\sqrt{2} \text{ ft}^2$
- e) none of the above

33. Evaluate  $\int_{-\infty}^{\infty} \frac{1}{x^2 - 6x + 10} dx$ .

- a) 0
- b)  $\frac{1}{2}$
- c)  $\infty$
- d)  $\pi$
- e) none of the above

34. The side of a cube is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, estimate the possible error in the calculation of the volume of the cube, based on this measurement.

- a)  $\pm 0.000001 \text{ in}^3$
- b)  $\pm 0.06 \text{ in}^3$
- c)  $\pm 0.027 \text{ in}^3$
- d)  $\pm 0.27 \text{ in}^3$
- e) none of the above

35. The product rule for differentiation gives the formula  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ . What is the analogous formula for the derivative of the product  $uvwz$  of four functions of  $x$ ?

- a)  $u \frac{dv}{dx} + v \frac{du}{dx} + w \frac{dz}{dx} + z \frac{dw}{dx}$
- b)  $uv \frac{dw}{dx} \frac{dz}{dx} + vw \frac{du}{dx} \frac{dz}{dx} + wz \frac{du}{dx} \frac{dv}{dx} + uz \frac{dv}{dx} \frac{dw}{dx}$
- c)  $uvw \frac{dz}{dx} + vwz \frac{du}{dx} + uwz \frac{dv}{dx} + uvz \frac{dw}{dx}$
- d)  $uv \frac{dz}{dx} + vw \frac{du}{dx} + wz \frac{dv}{dx} + uz \frac{dw}{dx}$
- e) none of the above

**Reminder**

**Question 36 will be used as a tie-breaker, if necessary.**

36. Find  $\lim_{x \rightarrow 1} \left[ \arcsin \left( \frac{\sqrt{4x-2}}{1-x} \right) \right]$ .

a)  $\frac{\pi}{2}$

b)  $-1$

c)  $-\frac{\pi}{2}$

d)  $\frac{1}{2}$

e) none of the above

37. Calculate  $\frac{d}{dx} \left[ \int_x^5 e^{t^2} dt \right]$  at  $x = 0$ .

a)  $-\frac{1}{2}$

b)  $\frac{1}{2}$

c) 1

d) -1

e) none of the above



38. Find  $\int \frac{dx}{\log_{\sqrt{x}} 2}$ .

a)  $\frac{x}{\log_{\sqrt{x}} 2} - \frac{x}{2 \ln 2} + C$

b)  $2x \log_{\sqrt{x}} 2 + C$

c)  $\frac{(2x \ln 2)^2}{\log_{\sqrt{x}} 2} + 2x \ln 2 + C$

d)  $\frac{2x}{\log_{\sqrt{x}} 2} - \frac{x}{\ln 2} + C$

e) none of the above

39. Find all the extrema of the function  $f(x) = x + \sin x$  on the interval  $[0, 2\pi]$ .

a)  $\left(\frac{3\pi}{2}, -1 + \frac{3\pi}{2}\right), (0, 0)$

b)  $(2\pi, 2\pi), (0, 0)$

c)  $(2\pi, 2\pi), (\pi, \pi)$

d)  $(\pi, \pi), (0, 0)$

e) none of the above

40. Given  $\int_{-2}^2 f(x) dx = 7$  and  $\int_0^2 f(x) dx = 11$ , calculate  $\int_0^{-2} 3f(x) dx$ .

a) -18

b) 12

c) 18

d) -12

e) none of the above