# *Twenty Third Annual*
# *University of North Georgia*
# *Mathematics Tournament*
# *April 1, 2017*

## *Morning Component*

***Good morning!***

***Please do NOT open this booklet until given the signal to begin.***

***There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.***

***The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 5 will be used again as a tie-breaker.***

***This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.***

***You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.***

***Good luck!***

## *Do Not Open Until Signaled.*

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**_Twenty Third Annual University of North Georgia_**
**_Mathematics Tournament_**

***You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.***

1. Find the limit: $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{\sin\left(\frac{1}{n}\right)}{n}}$ .

   a) Does not exist

   b) 1

   c) 0

   d) $\infty$

   e) None of the above

2. Two people start from the same point. One walks east at 3 mph and the other walks northeast at 2 mph. How fast is the distance between them changing after 15 minutes?

   a) $\sqrt{13-\sqrt{2}}$ mph

   b) $\sqrt{13-6\sqrt{2}}$ mph

   c) $\sqrt{13-5\sqrt{2}}$ mph

   d) $\sqrt{13-7\sqrt{2}}$ mph

   e) None of the above

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3. Find the limit: $\lim_{x\to 0^+} \frac{1}{\sin(x)} \int_0^{\sin(x)} e^{t^2}\,dt$ .

a) 0

b) $\infty$

c) 1

d) $-1$

e) None of the above

4. If $f'$ is continuous, $f(2)=0$, and $f'(2)=14$, find the limit:

$$\lim_{x\to 0} \frac{f(2+3x)+f(2+5x)}{x}$$

a) 100

b) $-5$

c) 5

d) 112

e) None of the above

**Reminder**

Question 5 will be used as a tie-breaker, if necessary.

5. Find the minimum value of the function $f(x)=\sin^6 x+\cos^6 x$ .

a) 0

b) $\frac{1}{4}$

c) $\frac{1}{2}$

d) $\frac{3}{4}$

e) None of the above

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6. Find the limit: $\lim\limits_{x\to 3} \dfrac{\sqrt{x+6}-3}{x^3-3x^2}$.

   a) $0$

   b) $-\dfrac{5}{54}$

   c) $\dfrac{1}{54}$

   d) $\dfrac{1}{3}$

   e) None of the above

7. For what values of $a$ and $b$ is the following equation true?

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{x^3}+a+\frac{b}{x^2}\right)=0$$

   a) $a=1, \quad b=\dfrac{1}{2}$

   b) $a=1, \quad b=-2$

   c) $a=\dfrac{4}{3}, \quad b=-2$

   d) $a$ and $b$ do not exist

   e) None of the above

8. Find the average value of the function $f(x)=\sin^4 x+\cos^4 x$ over the closed interval $[0, \pi]$.

   a) $\dfrac{3}{8}$

   b) $\dfrac{1}{2}$

   c) $\dfrac{5}{8}$

   d) $\dfrac{3}{4}$

   e) None of the above

9. Imagine that you increase the dimensions of a square with side $x_1$ to a square with side $x_2$. The change in the area of the square, $\Delta A$, is approximated by the differential $dA$. In this example, $dA$ is

   a) $(x_2 - x_1)\cdot 2x_1$
   b) $(x_2 - x_1)\cdot 2x_2$
   c) $x_2^{\,2} - x_1^{\,2}$
   d) $(x_2 - x_1)^2$
   e) None of the above

10. Find $y'$, if $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x$. Assume $x > 1$.

   a) $\dfrac{1}{2x\sqrt{x^2-1}}$
   b) $\dfrac{\pi}{2}$
   c) $\dfrac{x}{\sqrt{x^2-1}} - \dfrac{1}{x\sqrt{x^2-1}}$
   d) $0$
   e) None of the above

11. Find the limit: $\lim\limits_{n\to\infty}\left(\dfrac{n^2+n+3}{n^2+3n+5}\right)^n$.

   a) $\infty$
   b) $0$
   c) $\dfrac{1}{e^2}$
   d) $e^2$
   e) None of the above

12. If $f(x) = \sin x \cos x$, $f'(x) =$

a) $1 - 2\sin^2 x$

b) $2\cos^2 x + 1$

c) $2\cos^2 x - 2$

d) $\frac{1}{2}\cos(2x)$

e) None of the above

13. Find all possible functions $f(x)$ such that $\int_0^x \sqrt{1+f'(t)}\,dt = \frac{2}{3}x^{\frac{3}{2}}$, assuming $f'$ exists.

a) $f(x) = \frac{x^2}{2} - x + C$, where $C$ is a constant

b) $f(x) = \frac{x^2}{3} + x + C$, where $C$ is a constant

c) $f(x) = x - 1 + C$, where $C$ is a constant

d) $f(x) = x^2 + C$, where $C$ is a constant

e) None of the above

14. Find 2017th derivative of $\sin(2x)$.

a) $2^{2017}\sin(2x)$

b) $-2^{2017}\sin(2x)$

c) $2^{2017}\cos(2x)$

d) $-2^{2017}\cos(2x)$

e) None of the above

15. Find the limit: $\lim_{n\to\infty} \frac{8}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\sqrt{\frac{3}{n}}+\cdots+\sqrt{\frac{n}{n}}\right)$ .

a) $\frac{16}{3}$

b) 4

c) 16

d) $\frac{8}{5}$

e) None of the above

16. Find the coordinates of the point on the curve $y=\sqrt{x}$ that is closest to the point $(2,1)$.

a) $\left(\frac{3}{2},\frac{\sqrt{6}}{2}\right)$

b) $\left(\frac{2+\sqrt{3}}{2},\frac{1+\sqrt{3}}{2}\right)$

c) $\left(\frac{2-\sqrt{3}}{2},\frac{-1+\sqrt{3}}{2}\right)$

d) $\left(\frac{7-4\sqrt{3}}{4},\frac{-2+\sqrt{3}}{2}\right)$

e) None of the above

17. Find a function $f$ and a number $a$ such that

$$5+\int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}, \text{ for all } x>a>0.$$

a) $f(x)=\sqrt{x}, \quad a=1$

b) $f(x)=x^{3/2}, \quad a=\frac{25}{4}$

c) $f(x)=x^{4/3}, \quad a=1$

d) $f$ and $a$ do not exist

e) None of the above

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18. Find the limit: $\lim_{x\to 1} \frac{x-1}{\ln x}$.

(a) $-\infty$

(b) $-e^{-\pi}$

(c) 0

(d) 1

(e) None of the above

19. Evaluate the definite integral: $\int_0^{617\pi} |\sin(x)|\, dx$.

a) $123-617\pi$

b) 2468

c) 1234

d) $617\pi$

e) None of the above

20. Find the limit: $\lim_{x\to\infty} \left(\sqrt{2x^2+2x} - x\sqrt{2}\right)$

a) $\frac{1}{2}$

b) 1

c) Diverges to $+\infty$

d) $\frac{\sqrt{2}}{2}$

e) None of the above

21. Find the area of the surface swept out by revolving the circle of radius 1 unit, centered at the point $(0,1)$ in the $xy$-plane, about the $x$-axis.

a) $4\pi^2$ unit$^2$

b) $2\pi$ unit$^2$

c) $2\pi^2$ unit$^2$

d) $\left(\frac{\pi}{2}-1\right)^2$ unit$^2$

e) None of the above

22. Use implicit differentiation to find $y'$ if $x^2+y^2-3x+6y=9$.

a) $\frac{2x-3}{2y+6}$

b) $\frac{3-2x}{2y+6}$

c) $\frac{3-2x-2y}{6}$

d) $\frac{12-2x}{2y+6}$

e) None of the above

23. Evaluate the definite integral: $\int_{2012}^{2014}\left((x-2013)^2+(x-2015)^2\right)dx$.

a) 5145

b) $\frac{22}{7}$

c) $\frac{409}{608}$

d) $\frac{28}{3}$

e) None of the above

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24. Which of the following integrals will yield the volume generated by rotating the region bounded by $y = x^2$ and $y = 2 - x^2$ about the line $x = 1$?

a) $2\pi \int_{-1}^{1} (1-x)\left(2-2x^2\right) dx$

b) $2\pi \int_{0}^{1} \left(1-x^2\right)^2 dx$

c) $2\pi \int_{0}^{1} \left(\left(2-x^2\right)^2 - x^2\right) dx$

d) $2\pi \int_{-1}^{1} x\left(2-x^2\right) dx$

e) None of the above

25. Evaluate the definite integral: $\int_{0}^{\frac{\sqrt{2}}{2}} \frac{8x^2}{\sqrt{1-x^2}} dx$.

a) $\frac{\pi}{4}$

b) $\pi - 2$

c) $\pi + 2$

d) $\frac{2\pi + 1}{2}$

e) None of the above

26. Evaluate the definite integral: $\int_{0}^{1} e^{\sqrt{x}} dx$.

a) 0

b) $2e$

c) $e$

d) 2

e) None of the above

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27. Let $f(x) = x^4 - 4x^3 + 1$. Find the open interval(s) where the function is concave up.

(a) $(-\infty, 0)$

(b) $(2, \infty)$

(c) $(-\infty, 0) \cup (2, \infty)$

(d) $(0, 2)$

(e) None of the above

28. Evaluate the definite integral: $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{-3x^2 \sin(x)}{1 + x^6}\, dx$.

a) 0
b) 1
c) $\pi$
d) Does not exist
e) None of the above

29. Find the integral: $\displaystyle\int \frac{x}{1 + \sqrt{x-1}}\, dx$.

a) $(x-1)^{3/2} - x + 1 + \sqrt{x-1} - \ln\left(1 + \sqrt{x-1}\right) + C$

b) $(x-1)^{3/2} - x + 1 + 4\sqrt{x-1} - 4\ln\left(1 + \sqrt{x-1}\right) + C$

c) $\frac{2}{3}(x-1)^{3/2} - x + 1 + \sqrt{x-1} - 4\ln\left(1 + \sqrt{x-1}\right) + C$

d) $\frac{2}{3}(x-1)^{3/2} - x + 1 + 4\sqrt{x-1} - 4\ln\left(1 + \sqrt{x-1}\right) + C$

e) None of the above

30. The mean value theorem for integrals asserts that if $f$ is continuous on $[a,b]$, then there is $c$ in $[a,b]$ such that $f(c)$ is equal to the average value of $f$ on $[a,b]$. For the function $f(x)=\sqrt{4-x^2}$, find $c$ that satisfies the conclusion of the theorem on $[-2,2]$.

a) $c=\dfrac{\pi}{2}$

b) $c=\pm\sqrt{2-\dfrac{\pi}{2}}$

c) $c=\pm\sqrt{4-\dfrac{\pi^2}{4}}$

d) $c=\pm\sqrt{\pi^2-4}$

e) None of the above

31. Evaluate the definite integral: $\displaystyle\int_{2^5}^{3^5}\frac{1}{x-x^{3/5}}\,dx$. Leave your answer as a single logarithm.

a) $\dfrac{5}{2}\ln\left(\dfrac{8}{3}\right)$

b) $\dfrac{3}{2}\ln\left(\dfrac{5}{3}\right)$

c) $\dfrac{3}{5}\ln\left(\dfrac{8}{5}\right)$

d) $\dfrac{5}{3}\ln\left(\dfrac{8}{3}\right)$

e) None of the above

32. Find the area of the region enclosed by the curve $x^{2/5}+|y|=1$.

a) $\dfrac{2}{3}$

b) 1

c) $\dfrac{8}{7}$

d) 0

e) None of the above

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33. The density of oil in a circular oil slick on the surface of the ocean at a distance $r$ meters from the center of the slick is given by $\delta(r)=\dfrac{45}{1+r^2}$ kilograms per square meter. Find the exact value of the mass of the oil slick, if the slick extends from $r=0$ to $r=11$ meters.

a) $45\pi\ln(122)$ kg
b) $\dfrac{45}{2}\ln(122)$ kg
c) $\dfrac{495\pi}{61}$ kg
d) $45\arctan(122)$ kg
e) None of the above

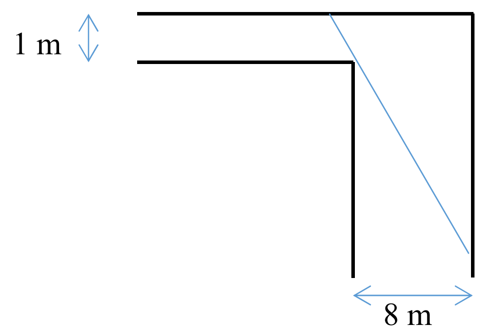
34. Evaluate the definite integral: $\displaystyle\int_{-\frac{2}{\pi}}^{0}\left(2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)\right)dx$.

a) $\pi^2$
b) $\pi$
c) $\dfrac{4}{\pi^2}$
d) The integral cannot be evaluated.
e) None of the above

35. Find the area of the region enclosed by the curves $y=x+3$ and $y=\left|x^2-4x+3\right|$.

a) 14
b) $\dfrac{56}{3}$
c) $\dfrac{88}{3}$
d) $\dfrac{109}{6}$
e) None of the above

36. The figure shows two corridors meeting at the right angle. One has a width 1 meter, and the other, width 8 meters. Find the length of the longest pipe that can be carried horizontally from one corridor, around the corner, and into the other corridor.




a) $4\sqrt{5}$ m
b) $5\sqrt{5}$ m
c) $5\sqrt{4}$ m
d) $4\sqrt{4}$ m
e) None of the above

37. If $x\sin(\pi x) = \int_0^{x^2} f(t)\,dt$, where $f$ is a continuous function, find $f(4)$.

a) $\frac{\pi}{2}$
b) $\frac{\pi}{4}$
c) $\pi$
d) $\frac{\pi}{3}$
e) None of the above

38. Find the integral: $\int \frac{\cos\theta}{\sin^2\theta + \sin\theta - 6}\,d\theta$.

a) $\ln\left|\sqrt[3]{(\sin\theta+2)/(\sin\theta-3)}\right| + C$
b) $\ln\left|\sqrt[3]{(\sin\theta-2)/(\sin\theta+3)}\right| + C$
c) $\ln\left|\sqrt[5]{(\sin\theta-2)/(\sin\theta+3)}\right| + C$
d) $\ln\left|\sqrt[5]{(\sin\theta+2)/(\sin\theta-3)}\right| + C$
e) None of the above

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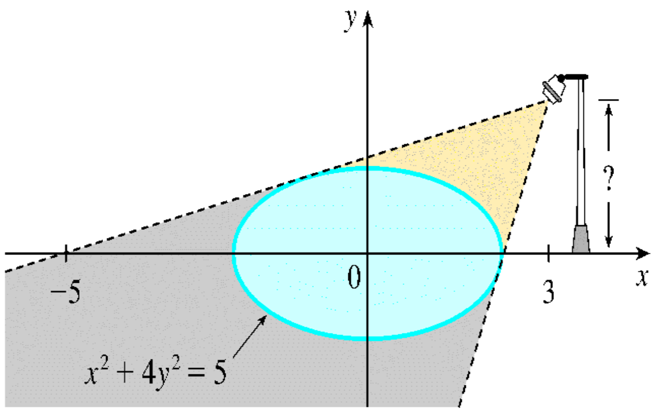
39. The figure below shows a lamp located three units to the right of the $y$-axis and a shadow created by the elliptical region $x^2 + 4y^2 \le 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the $x$-axis is the lamp located?




a) $2\sqrt{3}$

b) $4$

c) $\dfrac{4\sqrt{3}}{3}$

d) $2$

e) None of the above

40. Find an equation of the tangent line to the graph of $y = g(x)$ at $x = -1$, where

$$g(x) = 3 + \int_1^{x^2} \sec(t-1)\, dt$$

a) $y = 2x + 1$

b) $y = -2x + 1$

c) $y = 3 + \sec\left(x^2 - 1\right)$

d) $y = 3x + \sec\left(x^2 - 1\right)$

e) None of the above